Homework #4 (10 points) - Show all work on the following problems:

Problem 1 (3 points): Starting with the expression $\frac{dW}{dt} = \int(\vec{J_f} \cdot \vec{E})d\tau$ for the work done on free charges and currents by the electromagnetic fields, derive a new version of Poynting's theorem in matter, following the treatment in section 8.1.2. Show that the Poynting vector becomes $\vec{S} = \vec{E}\times\vec{H}$, and that the rate of change of the energy density in the fields is $\frac{\partial u}{\partial t} = \vec{E} \cdot \frac{\partial \vec{D}}{\partial t} + \vec{H} \cdot \frac{\partial \vec{B}}{\partial t}$. For linear media (so that E and D are proportional to each other, and B and H are proportional to each other), show that $u = \frac{1}{2}[\vec{E} \cdot \vec{D} + \vec{B} \cdot \vec{H}]$.

Problem 2 (2 points): Show that a standing wave $f(z,t) = A\, sin(kz) \cos{(kvt)}$ is a solution of the wave equation, and that it can be written as a sum of a wave traveling to the left and a wave traveling to the right.

Problem 3 (5 points):

3a (2 points): Derive the real electric and magnetic fields, in Cartesian components, for a monochromatic plane wave of amplitude E_0, frequency ω, and $\delta=0$, traveling in the $-x$ direction and polarized in the z direction. Explicitly write down the Cartesian coordinates for the wave vector $\vec{k}$ and the polarization vector $\hat{n}$. Finally, sketch the wave fields, in a format similar to Fig. 9.10 in the textbook.

3b (3 points): Do the same thing for a wave with the same parameters, but traveling in the direction from the origin to the point (1,1,1), with its polarization parallel to the x-z plane.